APPLICATION OF DERIVATIVES

Differentiability in an interval

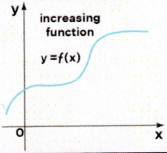
The rate of change of a quantity 'y' with respect to another quantity 'x' is known as the derivative

Monotonicity

Monotonic Increasing

$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$

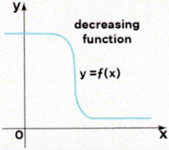
$\forall\, x_1, x_2 \in$ Domain



Monotonic Decreasing

$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$

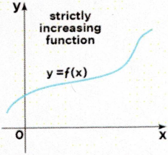
$\forall\, x_1, x_2 \in$ Domain



Strictly Increasing

$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

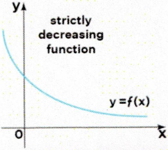
$\forall\, x_1, x_2 \in$ Domain



Strictly Decreasing

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

$\forall\, x_1, x_2 \in$ Domain



Method to Test Monotonicity

- Monotonic Increasing $\Rightarrow f'(a) > 0$
- Monotonic Decreasing $\Rightarrow f'(a) < 0$

At a Point

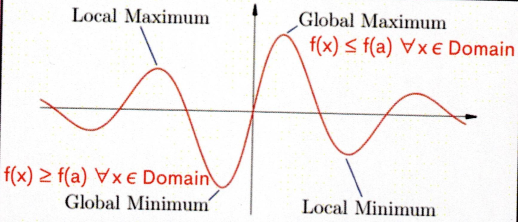
A function defined in the interval [a,b] ;
- Monotonic Increasing $\Rightarrow f'(x) \geq 0$
- Monotonic Decreasing $\Rightarrow f'(x) \leq 0$
- Constant $\Rightarrow f'(x) = 0$
- Strictly Increasing $\Rightarrow f'(x) > 0$
- Strictly Decreasing $\Rightarrow f'(x) < 0$

In an Interval

Monotonic Increasing	Monotonic Decresing	Non-Monotonic
x^3, $x.\|x\|$, $[x]$, e^x, tanx	$1/x$, $1-2x$, e^{-x}, cotx	x^2, $\|x\|$, $e^x + e^{-x}$, sinx

- If f(x) is strictly increasing function on an interval [a, b], then f^{-1} exists and it is also a strictly increasing.
- If f(x) is strictly increasing continuous function on an interval [a, b] , then f^{-1} is continuous on [f(a), f(b)]
- If f(x) and g(x) are monotonically (or strictly) increasing (or decreasing) function on [a, b], then gof (x) is a monotonically (or strictly) increasing function on [a, b]
- If one of the two functions f(x) and g(x) is strictly (or monotonically) increasing and other a strictly (monotonically) decreasing, then gof (x) is strictly (monotonically) decreasing on [a, b]
- Constant function is both mono ⬆ and mono ⬇.

Maxima & Minima



Extreme/Turning Points : The Points at which f(x) obtains local maxima/minima and value of both are called **Extreme values.**

- A fn may have more than one maxima/minima pts.
- A Minimum value may not be the least value and May be greater than some maximum value of the fn.
- In a continuous fn with only one maximum (minimum) pt., that pt. is the greatest (least) value.
- Monotonic fn do not have extreme points

Necessary Condition for maxima/minima at $x = a$ is $f'(a) = 0$ if it exists (but not sufficient)

Sufficient Condition : First Derivative Test at x = a

- Find dy/dx and Put it equal to 0
- Solve the equation for x and assume $c_1, c_2 \ldots$ as roots of equation.
- For $x = c_1$, if dy/dx changes sign from + to –, then fn obtains local maxima at $x = c_1$.

- For $x = c_1$, if dy/dx changes sign from - to +, then fn obtains local minima at $x = c_1$.
- if dy/dx doesn't change sign, Maxima or Minima Doesn't exist.
- The extreme value is given by $f(c_1)$

Sufficient Condition : Higher Derivative Test

- f(a) is maximum if $f'(a) = 0$ and $f''(a) < 0$
- f(a) is minimum if $f'(a) = 0$ and $f''(a) > 0$
- if $f'(a) = 0$; $f''(a) = 0$ & $f'''(a) \neq 0$, then x = a is not the extreme point of the function.
- If $f'(a) = f''(a) = f'''(a) = ... = f^{n-1}(a) = 0$ and $f^n(a)$ exists
  - n is even, $f^n(a) < 0$; Point of Local Maximum
  - n is even, $f^n(a) > 0$; Point of Local Minimum
  - n is odd ; neither maximum or minimum

Concace up graph	Concave down graph
$f''(x) > 0$	$f''(x) < 0$
for $x_1, x_2, x_3 \in (a,b)$	
$f\left(\frac{x_1 + x_2}{2}\right) \leq \frac{f(x_1) + f(x_2)}{2}$	$f\left(\frac{x_1 + x_2}{2}\right) \geq \frac{f(x_1) + f(x_2)}{2}$

- Maxima and Minima occur alternatively
- if f(x) is maxima at a point then 1/f(x) will be minima.
- If $f(x) \to \infty$ as$\to$a or b and $f'(x) = 0$ only for one value of x (say c) between a and b, then f(c) is necessarily the minimum and the least

Equation of Tangents and Normals, $m = \tan\theta$

Slope at point (x_1, y_1) can be written as $\frac{dy}{dx} = m$

• Equation of **Tangent** at (x_1, y_1)	$(y - y_1) = \frac{dy}{dx}(x - x_1)$
• Equation of **Normal** at (x_1, y_1)	$(y - y_1) = \frac{-1}{dy/dx}(x - x_1)$

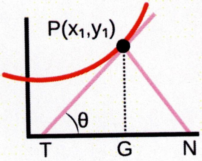
Length of Tangents and Normals

PT = Tangent $= |y_1|\sqrt{1 + \frac{1}{m^2}}$
(GP.cosecθ)

PN = Normal $= |y_1|\sqrt{1 + m^2}$
(GP.secθ)

TG = Subtangent $= \frac{|y_1|}{m}$
(GP.cotθ)

GN = Subnormal $= |y_1 m|$ = **(GP.tanθ)**



Angle of Intersection between two curves

m_1 and m_2 are the slopes of the tangents T_1 and T_2 at the intersection point (x_1, y_1).

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$